Energy-efficient Team Orienteering Problem in the Presence of Time-Varying Ocean Currents

Ariella Mansfield\textsuperscript{1}, Douglas G. Macharet\textsuperscript{2}, and M. Ani Hsieh\textsuperscript{1}

Abstract— Autonomous Marine Vehicles (AMVs) have gained interest for scientific and commercial applications, including pipeline and algae bloom monitoring, contaminant tracking, and ocean debris removal. The Team Orienteering Problem (TOP) is relevant in this context as Multi-Robot Systems (MRSs) allow for better coverage of the area of interest, simultaneous data collection at different locations, and an increase in the overall robustness and efficiency of the mission. However, route planning for AMVs in dynamic ocean environments is challenging due to the coupling of environmental and vehicle dynamics. We propose a multi-objective formulation that accounts for the trade-offs between visiting multiple task locations and energy consumption by the vehicles subject to a time budget. This work focuses on vehicles that can maintain a constant net speed but can be adapted to vehicles with constant thrust. Different from existing approaches, our method is able to leverage time-varying ocean currents to improve the energy efficiency of resulting routes. We validate our approach experimentally by superimposing ocean flow models with benchmark instances of the TOP.

I. INTRODUCTION

Autonomous Marine Vehicles (AMVs) are becoming increasingly popular in various applications, such as oceanographic research, underwater archaeology, and maritime surveillance. However, there are several challenges involved in the use of such vehicles, including their costs, environmental dynamics, energy consumption, and regulatory requirements. Moreover, environmental changes, such as weather conditions and water currents, can significantly impact the performance and reliability of these vehicles.

In this context, routing problems play a crucial role in the successful deployment and operation of autonomous vehicles. These problems involve finding the most efficient tour to sequentially visit distinct sites, taking into account factors such as energy consumption, time constraints, and environmental conditions. The Orienteering Problem (OP)\cite{1} is particularly relevant, since it allows to model more realistic aspects such as locations associated with specific rewards and vehicles with limited travel budgets. The OP is NP-hard\cite{1}, since it is a generalization of the Travelling Salesperson Problem (TSP).

In real-world scenarios, some environmental features can greatly impact the tour, e.g., ocean and/or atmospheric currents. In practice, in the ocean environment there is a trade-off between the expected utility (reward) of a task and the cost of travelling to its location.

Therefore, in a marine environment, we are interested in efficiency in terms of time and energy. In our previous work\cite{2}, we proposed a multi-objective formulation to tackle the OP in the presence of static flow fields. We focus on vehicles that can maintain a constant net speed, but our method could be extended to vehicles that output constant thrust. In this paper, we extend the previous approach to consider multi-robot teams and time-varying flows. An illustration depicting an example scenario is depicted in Figure 1.

Our main contributions are:

- The formulation of a multi-objective optimization problem for a heterogeneous team operating in a known, time-varying vector field. This formulation accounts for the trade-offs between energy expenditure and reward collection, subject to a mission budget.
- An evolutionary algorithm is proposed to solve this formulation. By combining the task and path planning problems into one problem, our method is able to deal with the coupling between the environmental dynamics and the costs of operating the vehicles.
- We present experimental results from both a simulated ocean environment and nowcast data from the NOAA’s Global Real-Time Ocean Forecast System (RTOFS)\cite{3} database.

II. RELATED WORK

Motion planning is crucial for autonomous mobile robots operating in a diverse set of environments. Most existing approaches focus on finding paths that are optimized considering the path’s length or traversal time. In practice,
these methods should also consider other important factors, such as energy expenditure, especially when vehicles must operate in complex environments with significant constraints on resources and little opportunity for human intervention.

In dynamic environments like the ocean and the atmosphere, the environmental forces experienced by the vehicle are inherently coupled with its dynamics. This coupling can make path planning harder but opens an opportunity for the vehicle to take advantage of these forces to compute more energy efficient paths. Different works have tackled this problem by considering a single point-to-point path in environments with time-invariant [4], [5] or time-varying [6]–[9] flow fields. Path planning where forecasting uncertainties are taken into account has also been developed by [10], [11]. Others have leveraged the global topology of the fluid flow vector field to find energy efficient trajectories [12], [13].

However, the sequential visiting of multiple sites imposes new challenges. The OP [1] is part of a broad class of routing problems where the vehicle has a limited budget and must visit various task locations, each with an associated reward. The goal is to determine the optimal sequence of visits that maximizes the rewards collected. Since a budget constraint must be respected, it might be necessary to select a subset of the locations to be visited. It has been generalized in many different ways to consider characteristics such as motion and exposure constraints [14], [15], correlated and time-varying profits [16], [17], fault-tolerance [18], among others. The task allocation problem in an ocean environment has been explored in [19], [20], however these works do not consider time-varying flow fields and optimize for a single objective.

In our previous work [2], we proposed a multi-objective formulation for the OP to be applied in fluid environments. In addition to maximizing the collected reward, the goal was to minimize the energy expenditure by leveraging the environmental dynamics. For Multi-Robot Systems (MRS), determining the order and which robot will visit each location is related to a task planning phase, and there is another step which is to determine the paths between these tasks. The Team Orienteering Problem (TOP) [21] is the multi-robot variant of the classic OP formulation, where robots must uniquely visit a disjoint set of locations. In [22], we proposed a planning and scheduling algorithm for a TOP variant where the collected reward at each location is related to the service time each robot spends attending a demand.

In this paper, we consider the use of multiple, heterogeneous vehicles in the presence of time-varying environmental flows. We formulate it as a multi-objective optimization problem and use an evolutionary algorithm to solve the task and path planning problems in a combined manner.

III. PROBLEM FORMULATION

A. Vehicle and Environment Model

We consider a 2-D representation $\mathbb{W} \in \mathbb{R}^2$ for modeling a marine environment. Predictive and descriptive models of these currents have been studied and used for different regions of the world [3], [23], [24]. Since ocean currents impact the movement of vehicles along a given set of trajectories, we employ a kinematic representation of the vehicle and thus for a given point $q \in \mathbb{W}$, at time $t$, the net velocity of the vehicle $V_{net}$ is a result of the thrust vector $V_u$ and the ocean currents $V_c$ given by

$$ V_{net}(q,t) = V_c(q,t) + V_u(q,t). \quad (1) $$

Since vehicles use propulsion for thrust to overcome drag forces, the energy expenditure is given by

$$ E(\Gamma(t)) = \int_t^\infty k_d V_u^2(\Gamma(t)) dt, \quad (2) $$

where $\Gamma(t)$ is the path travelled, whose time derivative is the net velocity $V_{net} = d\Gamma(t)$. $k_d$ is the drag coefficient, $V_u$ is the speed as a result of thrust, and the drag model $\alpha \in \{2,3,\ldots\}$ ($\alpha = 2$ for linear drag, $\alpha = 3$ for quadratic drag, and so on) [8]. Note that the energy cost here is impacted by the environmental currents, as (1) gives the relationship between $V_{net}$, $V_u$, and $V_c$.

B. Unified Task and Path Planning Problem

Let $\mathcal{N} = \{n_1, \ldots, n_N\}$ be the set of spatially distributed node locations with $n_i \in \mathbb{W}$ and let $\mathcal{A} = \{a_1, \ldots, a_M\}$ be a team of cooperative vehicles. Each location is associated with a known positive profit $r_i$ which can be collected at the node.

**Problem 1** (Energy-efficient TOP with Time-Varying Ocean Currents). The aim is to maximize the total rewards collected and minimize energy expenditure, i.e.:

$$ \max_{\Gamma_{ijm}} \sum_{i=1}^{N-1} \sum_{m=1}^{M} r_{iym}, \quad (3a) $$

$$ \min \sum_{i=1}^{N-1} \sum_{j=2}^{M} x_{ijm} E(\Gamma_{ijm}), \quad (3b) $$

where $\Gamma_{ijm}$ is a decision variable representing the path from $n_i$ to $n_j$ by vehicle $a_m$. $E(\Gamma_{ijm})$ is the energy given by (2). $x_{ijm}$ is a binary decision variable that is equal to 1 when vehicle $a_m$’s route includes the arc $(n_i,n_j)$ and 0 otherwise.

$y_{ijm}$ is a binary decision variable that gets a value of 1 when $n_i$ is included in $a_m$’s route and 0 otherwise.

This optimization is subject to the following constraints:

$$ \sum_{j=2}^{M} x_{ijm} = \sum_{i=1}^{N-1} x_{iNm} = M \quad (4a) $$

$$ \sum_{i=1}^{N-1} x_{ikm} = \sum_{j=2}^{N} x_{kjm} = y_{km}; \quad k=2,\ldots,(N-1) \quad m=1,\ldots,M \quad (4b) $$

$$ \sum_{m=1}^{M} y_{km} \leq 1; \quad k=2,\ldots,(N-1) \quad m=1,\ldots,M \quad (4c) $$

$$ \Gamma_{ijm}(t^{ijm}) = n_i; \quad \Gamma_{ijm}(t^{ijm}) = n_j; \quad 0 \leq t^{ij} \leq t^{ij}, \quad i,j = 1,\ldots,N \quad m=1,\ldots,M \quad (4d) $$
Our MOGA is shown in Algorithm 1.

\[
\sum_{i=1}^{N-1} \sum_{j=2}^{N} x_{ijm} \int_{\Gamma_{ijm}} \frac{1}{\| V_{net}(q,t) \|} \, dt \leq T_{max,m} \\
\forall m = 1, \ldots, M 
\]  

(4e)

\[2 \leq u_{im} \leq N; \quad \forall i = 2, \ldots, N \quad \forall m = 1, \ldots, M \]  

(4f)

\[u_{im} - u_{jm} + 1 \leq (N - 1)(1 - x_{ijm}); \quad \forall i, j = 2, \ldots, N \quad \forall m = 1, \ldots, M \]  

(4g)

Constraint (4a) ensures the routes start and end from the depots \( n_1, n_N \), respectively. Constraint (4b) ensures the connectivity of the routes. Constraint (4c) ensures that nodes are visited at most once. Constraint (4d) ensures that vehicle \( a_m \)'s path, \( \Gamma_{ijm} \), starts from \( n_i \) at some start time \( t_{ijm} \) and reaches \( n_j \) at \( t_{ijm} \). Constraint (4e) ensures that the vehicles adhere to the time budget \( T_{max,m} \). The variables \( u_{im} \) represent the relative positioning of \( n_i \) in vehicle \( a_m \)'s route, when selected for inclusion. Using these variables, constraints (4f)-(4g) enforce subtour elimination, following the Miller-Tucker-Zemlin formulation introduced in [25].

IV. METHODOLOGY

Genetic Algorithms (GAs) are a type of optimization algorithm inspired by the principles of natural selection. In GAs, a population of potential solutions is iteratively evolved through genetic operators such as selection, crossover, and mutation. The main advantages of GAs include their ability to find good solutions in large and complex search spaces, handle non-linear and non-convex objective functions, and consider different constraints.

However, classic GAs have some limitations, such as their inefficiency for problems with a large number of decision variables. To address this, Multi-Objective Genetic Algorithms (MOGAs) have been developed, which consider multiple conflicting objectives simultaneously. MOGAs have been widely applied in fields such as scheduling, design optimization, and control. In our formulation, no objective is given a preference over the others, rather we return the non-dominated front of individuals. An overview of the steps of our MOGA is shown in Algorithm 1.

Algorithm 1 Multi-Objective Genetic Algorithm

\[
\begin{align*}
1: & \text{Generate initial population} \\
2: & \text{Evaluate fitness of current individuals} \\
3: \text{while stopping condition is not satisfied do} \quad & \text{Iteratively select pairs of parent individuals} \\
4: & \quad \text{Generate new individuals by crossover/mutation} \\
5: & \quad \text{Evaluate fitness of new individuals} \\
6: & \quad \text{Replace current individuals with new ones} \\
7: \text{end while} \quad & \text{return} \quad \text{Return set of non-dominated individuals}
\end{align*}
\]

A. Encoding and Offspring Generation

The solutions in a GA are given by individuals in a population. Each individual is represented by a chromosome composed of genes. In our case, the chromosome, as depicted in Figure 2, is constructed as a matrix, with the genes given by the elements of the matrix. Row indices represent the \( N \) task locations and column indices represent each of the \( M \) vehicles.

Each gene contains information about the ordering \( \pi_{im} \), selection \( \xi_{im} \) and control points \( CP_{im} \) of the solution. The ordering parameter \( \pi_{im} \in (0,1) \) uses a random-key scheme similar to [26] to determine the relative position of node \( n_i \) in the tour for route \( a_m \). The binary selection parameter \( \xi_{im} \in \{0,1\} \) determines whether node \( n_i \) will be included in the route of \( a_m \). The set of control points \( CP_{im} \) are used to define a smooth B-spline path that goes through \( n_i \) for \( a_m \), specifying different control points for paths that are incoming and outgoing from the node. For additional details on these parameters, their initialization, offspring generation, and fitness evaluation we refer the reader to our previous work on the static, single vehicle case [2].

![Fig. 2: Representation of an individual’s chromosome with \( N \times M \) genes, each defined by \( \{ \pi_{im}, \xi_{im}, CP_{im} \} \).](image)

In the offspring generation step, we modified the process of mutating control points relative to our previous work in order to accommodate a greater diversity of test instances. Control points on a certain curve are mutated by adding a random value \( \Delta \in \mathbb{R}^2 \) to a given control point \( cp \). The movement \( \Delta \) is sampled using the beta distribution, Beta\((\alpha, \beta)\). The beta distribution was chosen for its definition on a closed interval, which allows us to limit the largest move allowed, and have parameters that allow us to manipulate the shape of the distribution.

Finally, after a new individual is generated, we execute a validation step to guarantee that all of its tours respect the budget constraint. If a tour’s travel time is above \( T_{max,m} \), we repeatedly and randomly remove nodes \( \xi_{im}(=0) \) from it until its travel time adheres to the budget. We ensure the smoothness of the tour by enforcing co-linearity conditions on a subset of the control points between B-spline curves.

B. Evaluation on a Time-Varying Vector Field

The fitness of an individual is related to both objective functions, i.e., one that is defined as the sum of the rewards collected at the visited nodes and the second defined as the energy expenditure along the vehicle’s tour.
Similar to [6], we assume that the net speed $\|V_{\text{net}}\|$ is constant in order to reduce the search space. Modifications could be made to accommodate an assumption of constant thrust $\|V_u\|$ following methodology laid out in [20]. These assumptions impart an implicit time-parametrized trajectory to the paths defined by a chromosome’s B-splines. Using this time-parametrized trajectory, we are able to evaluate the energy expenditure along the route in a known time-varying vector field $V_c(q, t)$.

V. EXPERIMENTS

We tested our method using models of ocean currents based on wind-driven double gyre models [27], Lamb vortices [24], and real-world nowcast predictions of ocean currents from NOAA’s Global Real-Time Ocean Forecast System (RTOFS) [3]. In the subsequent section, the wind-driven double gyre model is given by

$$V_{x}(q, t) = -\pi A \sin(\pi f(x, t)) \cos(\pi y)$$
$$V_{y}(q, t) = \pi A \cos(\pi f(x, t)) \sin(\pi y) \frac{\partial f}{\partial x}$$
$$f(x, t) = \epsilon \sin(\omega t) x^2 + (1 - 2 \epsilon \sin(\omega t)) x.$$  

Table I summarizes the GA parameters used in the experiments. We implemented the algorithm on Python 3.7 and used the DEAP library [28].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>200</td>
</tr>
<tr>
<td>Number of generations</td>
<td>200</td>
</tr>
<tr>
<td>Selection method</td>
<td>NSGA-III [29]</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.8</td>
</tr>
<tr>
<td>Crossover operator</td>
<td>Two-point crossover</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.6</td>
</tr>
<tr>
<td>Mutation probability (gene)</td>
<td>0.08</td>
</tr>
<tr>
<td>Beta Distribution ($\alpha, \beta$)</td>
<td>(3,4)</td>
</tr>
</tbody>
</table>

1) Energy Efficiency and Heterogeneous Teams: In an optimal solution, vehicles will find energy efficient paths relative to the ocean currents when travelling between tasks. Vehicles can leverage the environment to increase their energy efficiency by aligning their path with the ocean currents when possible, thereby reducing drag forces.

Fig. 3: A heterogeneous team in a static flow field.

We can see this effect clearly by examining Figure 3. In this example, we consider a static field using the wind-driven gyre model given by (5) with current strength $A = 0.5$ and time-varying amplitude $\epsilon = 0$. Two heterogeneous vehicles, with drag coefficients of $k_{d,1} = 0.1, k_{d,2} = 0.5$ and $\|V_{\text{net}}\| = 0.6$ are deployed. We see that the ordering of visits and the paths between them conform to the currents. The distribution of rewards relative to the vehicle budget allows for one vehicle to follow a path along the stable and unstable manifolds of the left gyre boundary while the other vehicle collects the rewards along the other gyres. Note that the high drag vehicle’s path followed the gyre boundary while the low drag vehicle collected the remaining rewards. This result aligns with the conclusions from [12], where they show that energy optimal paths in flows follow the Lagrangian Coherent Structures, an extension of invariant stable and unstable manifolds for time-dependent flows.

2) Impact of Time with Ocean Currents Nowcast Data: Our method can be used to find routes within a time-varying field, as illustrated in Figure 4. Time-varying ocean currents were collected from the RTOFS database for the period of November 9-16, 2021 in the North Atlantic Ocean. A pair of heterogeneous vehicles, with $T_{\text{max}}^1 = 83.3 hr, T_{\text{max}}^2 = 55.5 hr$, and $\|V_{\text{net}}\| = 0.6$ were deployed to a region with uniformly distributed task locations. The frames shown portray a solution on the non-dominated front obtained by our method at selected timesteps.

We observe that the vehicles’ paths tend to align with the field at the given timestep, thereby reducing drag forces. However, this objective is balanced by the attempt to maximize reward and visit additional locations. Additionally, since the vehicles are operating at a constant net speed, they continue moving at that speed even when adaptations, such as waiting for favorable conditions, could enable them to take advantage of subsequent currents and increase efficiency.

Fig. 5: Comparison of time-varying informed and static informed solutions.

We examine the impact of using a method that is informed by the full time-varying flow field in Figure 5. The results represent solutions to on the same test case shown from
Fig. 4: Solution of two vehicles in a time-varying field based on ocean currents in the North Atlantic Ocean at different timesteps. The title shows the Reward (R) and Energy (E) collected by the team. The legend shows the heterogeneous characteristics of the vehicles (budget $T_{\text{max}}$) and the breakdown of the reward and energy collected by each vehicle. In the final frame the length (l) and time (t) of each vehicle’s route is added to the legend.

Figure 4, over 20 trials. Static informed solutions were run on a GA that assumed the field remained constant from the first timestep. The points shown reflect the average energy and standard deviation for the rewards collected by individuals along the non-dominated front. The highest levels of reward were not reached in all trials, and the number of trials contributing is reflected in the color of the marker. From this plot, it is clear that the time-varying informed solutions, on average, are able to collect the same amount of rewards using less energy. Additionally, the standard deviation of the solutions is lower. The increased standard deviation in the energy for the static informed results can be explained by noting that these solutions were not solving with complete information of the currents. Since they were optimizing on a flow field that did not include the changes to the field over time, we expect there to be more variation in the actual energy that would be expended following these routes.

3) Teams: To test that our method on teams of vehicles we took the datasets from [21] and examined their performance when we combined them with a background flow field of the double gyre model (5). The workspace variables $x, y$ were transformed to scale the traditional domain of the double gyre model $[0, 2] \times [0, 1]$ to the domain defined by the task locations in the dataset. An example of one of these test cases can be seen in Figure 6, where the double gyre parameters used are: $A = 1, \epsilon = 0.5, \omega = \frac{\pi}{17.5}$, and the vehicle parameters are: $k_d = 0.1, \|V_{\text{net}}\| = 1, T_{\text{max}} = 17.5s$.

The difference between our results and those returned by [21] extend beyond the incorporation of information about the background currents. Our solutions impose smoothness constraints using B-splines, which means that paths between nodes found by our method are not as direct. Additionally, our solution optimizes a multi-objective, therefore we return a family of solutions and not just the time-optimal one.

VI. CONCLUSION AND FUTURE WORK

Routing problems, such as the Orienteering Problem (OP), are crucial for finding tour routes while considering factors such as energy consumption and environmental conditions. However, in real-world scenarios, features such as ocean and atmospheric currents can greatly impact the tour, and efficiency in terms of time and energy is critical especially in a marine environment.

In this paper, we propose a multi-objective formulation for a heterogeneous team operating in a time-varying flow that accounts for energy expenditure, reward collection, and mission budget trade-offs. We propose the use of an evolutionary algorithm to solve this problem by combining the task and path planning problems. Results considering simulated and real-world ocean data show that our approach is able to efficiently handle the coupling between the environmental dynamics and vehicle operation costs.

Future work could explore the introduction of vehicles with non-constant velocity in order to obtain more realistic solutions that more effectively utilize the flow. Another subsequent research direction would be to evaluate nodes with time-varying rewards, where it might be favorable for
the robot to take a longer path between adjacent locations in order to reach a site with a higher expected reward.

REFERENCES


Fig. 6: Team of vehicles set in a time-varying double gyre field.