Graph Embedding for the Division of Robotic Swarms

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Abstract— A key decision in human-swarm teaming is dividing a swarm into sub-swarms to address separate issues or to accomplish a task over a large area. As the swarms grow in complexity, the cognitive load needed to manually divide the swarm grows in magnitude. We propose a new multimodal graph embedding method to construct a unified representation that fuses multiple information modalities to describe and divide a swarm. Our approach takes into account diverse relationships in the swarm, such as spatial relationships, communication capabilities, and hierarchical structures. The relationship modalities are encoded as directed graphs which are embedded into a unified representation for each swarm agent. Experimental results show that our method successfully decides correct sub-swarms based on swarms' multifaceted internal structures, and outperforms baseline methods.

I. INTRODUCTION

Because of their robustness and flexibility, robotic swarms are being increasingly researched and used in large-scale applications, such as search and rescue and area exploration [1]. However, as the number of robots in a swarm increases, the swarms become cognitively more difficult for humans to understand and command [2]. At scale, the complexities of internal relationships become difficult for human operators to conceptualize. Figure 1 provides an illustration of how robots can appear organized in physical space, but also contain hierarchical relationships or communication capabilities within the swarm that are more difficult to perceive. These relationships are further complicated by swarm member interactions with obstacles and the surrounding environment. When combined, these challenges result in swarm states that are both difficult for a human operator to accurately perceive and for a system to display.

To address these problems, we propose a novel multimodal graph embedding approach to encode diverse relationships of robots in a swarm as graphs and integrate the multiple graphs into a unified representation that is applied to divide a swarm into sub-swarms, without the intervention of a human operator. We model each internal relationship of the robots in a swarm using a directed graph as an information modality. Given a set of member relationships, we construct multiple graphs that are applied as the input to our approach. Then, we propose a new multimodal Katz index to integrate multiple graphs of robot relationships and embed them into a unified representation for each robot in a swarm. Then, the
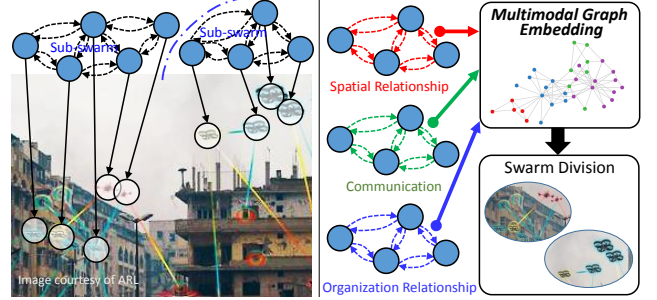
Fig. 1. A motivating example of automatic swarm division and our solution based on multimodal graph embedding. In real-world swarm systems, robot members in a swarm typically have a variety of relationships, such as spatial relationships, communication connectivity, and organization hierarchy. This complexity makes swarm division a difficult or impossible problem for a human operator. Our proposed multimodal graph embedding approach can integrate multiple relationship graphs and identify effective sub-swarms.

constructed representation is used to identify subdivisions of a swarm based upon unsupervised learning. Our multimodal graph-embedded swarm division is capable of fusing diverse swarm member relationships and identifying divisions without requiring explicit knowledge of tasks.

II. APPROACH

In real-world swarm deployment, members within a swarm typically have a multiple various relationships (e.g., spatial relationships, communication connectivity, and organization hierarchy). We encode the swarm with M graphs, where $\mathcal{G}_m$ is the graph describing the m-th relationship of swarm members. Each graph $\mathcal{G}_m$ is described by an adjacency matrix $\mathbf{A}_m \in \mathcal{R}^{N \times N}$, where each element a_{ij} is the weight of the edge connecting vertex v_i to vertex v_j.

To achieve our objective of encoding multiple graphs and embedding them into a single vector representation, we propose a new multimodal formulation of the Katz index [3] that is able to take multiple graphs as the input modalities and form a single similarity matrix $\mathbf{S} \in \mathcal{R}^{N \times N}$ that integrates the information of all graphs. To do this, we introduce a weight w_m for each $\mathbf{A}_m$ describing the importance of the relationship encoded by $\mathcal{G}_m$, where $\sum_{m=1}^{M} w_m = 1$. We then construct the multimodal similarity matrix $\mathbf{S}$ that embeds information of all graphs as follows:

$$\mathbf{S} = \left(\mathbf{I} - \alpha \sum_{m=1}^{M} w_m \mathbf{A}_m \right)^{-1} - \mathbf{I} \qquad (1)$$

In order to create a lower-dimensional representation than $\mathbf{S}$, which is necessary when embedding big graphs of a large-scale swarm, we perform Singular Value Decomposition: $\mathbf{S} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$. To further reduce the dimensionality of the

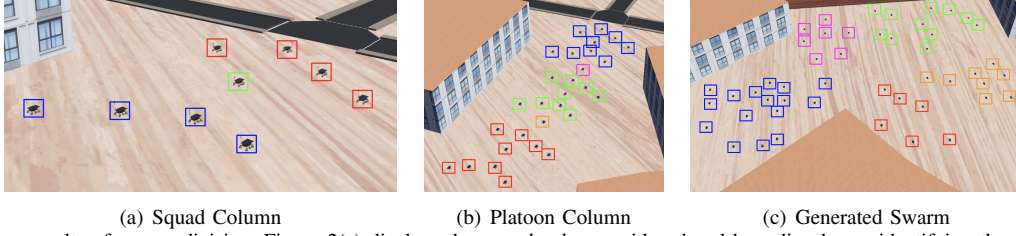(a) Squad Column   (b) Platoon Column   (c) Generated Swarm

Fig. 2. Qualitative results of swarm division. Figure 2(a) displays the *squad column*, with colored bounding boxes identifying the correct division into three clusters, consisting of the first team, the squad leader, and the second team [4]. Figure 2(b) displays the *platoon column*, displayed with the correct division into five clusters, consisting of the first squad, the platoon leader, the second squad, the platoon sergeant, and the third squad. Figure 2(c) displays a simulated swarm of 50 agents, divided into five sub-swarms.

representation, given a desired dimensionality D, we propose to use the first K left singular vectors and the first K right singular vectors to approximate $\mathbf{S}$, where $K = D/2$. This results in $\mathbf{U} \in \mathcal{R}^{N \times K}$ and $\mathbf{V} \in \mathcal{R}^{N \times K}$.

Finally, we construct the final representation matrix $\mathbf{X} \in \mathcal{R}^{N \times D}$ for all N swarm members by concatenation: $\mathbf{X} = (\mathbf{U}, \mathbf{V})$. where each row $\mathbf{x}_n \in \mathcal{R}^{1 \times D}$ of $\mathbf{X}$ represents the n-th agent of the swarm. Given this final representation, we apply unsupervised clustering to separate the swarm into sub-swarms.

III. EXPERIMENTS

We evaluated our approach on simulated robotic swarms in the Webots simulator, using expert-defined teams with known divisions and large-scale simulated swarms without ground truth. Each swarm was defined with graphs representing spatial positions, communication connectivity, and a set hierarchy. For quantitative evaluation, we present the clustering accuracy when dividing the expert-defined teams and utilize silhouette scores [5] for all sub-swarm divisions. Silhouette scores rate the quality of a clustering, with values closer to 1 being better and values closer to -1 being worse. To validate the superior performance of our approach, we compare it with baseline methods, including graph embedding approaches such as High-Order Proximity preserved Embedding (HOPE) [6].

We first evaluate our approach on the expert-defined team formations known as the *platoon column*, *platoon wedge*, and *platoon vee* and the *squad column*, *squad file*, and *squad line*, based on the field operations teaming protocol in [4]. This protocol defines correct sub-divisions for these formations. Platoon formations incorporate three squads and two separate leadership agents. Squad formations incorporate two teams and one separate leadership agent. Figures 2(a) and 2(b) displays the *squad column* and *platoon column* in the Webots simulator, with correct sub-divisions labeled.

TABLE I

COMPARISON OF ACCURACY AND SILHOUETTE SCORES

Method	Accuracy	Avg. Silhouette Score
HOPE [6] (Spatial)	85.62%	0.455
HOPE (Connectivity)	50.33%	0.018
HOPE (Hierarchy)	39.22%	0.167
Our Approach	**96.73%**	**0.680**

Out of a possible 306 agents in the six different formations, our approach clusters 96.73% of the agents correctly.

The best of the baseline methods is the HOPE embedding of the spatial relationships, clustering only 85.62% of agents correctly, showing that our method outperforms existing graph embedding methods. Using the silhouette score metric, our approach performs best with an average score of 0.680. Again, the highest existing method is the HOPE embedding of the spatial relationships, scoring 0.455. We note that our approach achieves its best results on the platoon formations, which contain over three times as many agents as the squad formations, suggesting that our approach's performance will extend to larger swarms. Table I shows that a linear relation exists between accuracy and silhouette score, validating the silhouette score as a metric for dividing large-scale simulated swarms without ground truth divisions.

To evaluate the effectiveness of our approach on a larger scale, we simulated larger multi-robot swarms consisting of 10 to 50 robots. These large-scale swarms have the same relationship modalities as the expert-defined teams, based on their generated positions. Figure 2(c) displays a simulated swarm of 50 agents in Webots. We repeatedly generated these large multi-robot systems evaluating each combination of swarm size and number of clusters 100 times. Our approach achieves the highest silhouette score of 0.676, beating the top baseline score of 0.427. This is consistent with our score of 0.680 on the expert-defined teams, where our approach achieved the highest clustering accuracy, suggesting that our identified sub-divisions of large-scale simulated swarms outperform the divisions identified by other methods and can identify useful swarm divisions without the intervention of a human operator.

REFERENCES

[1] B. Bayat, N. Crasta, A. Crespi, A. M. Pascoal, and A. Ijspeert, "Environmental monitoring using autonomous vehicles: a survey of recent searching techniques," *Current Opinion in Biotechnology*, vol. 45, pp. 76–84, 2017.

[2] G. Bales and Z. Kong, "Neurophysiological and behavioral studies of human-swarm interaction tasks," in *IEEE International Conference on Systems, Man, and Cybernetics*, 2017.

[3] L. Katz, "A new status index derived from sociometric analysis," *Psychometrika*, vol. 18, no. 1, pp. 39–43, 1953.

[4] Department of the Army, *U.S. Army Field Manual 3-21.9*, 2017.

[5] P. J. Rousseeuw, "Silhouettes: a graphical aid to the interpretation and validation of cluster analysis," *Journal of Computational and Applied Mathematics*, vol. 20, pp. 53–65, 1987.

[6] M. Ou, P. Cui, J. Pei, Z. Zhang, and W. Zhu, "Asymmetric transitivity preserving graph embedding," in *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2016.