2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
September 24–28, 2017, Vancouver, BC, Canada

Cooperative Transport of a Buoyant Load: A Differential Geometric Approach

Hadi Hajieghrary[1] and Dhanushka Kularatne[1], and M. Ani Hsieh [2]

***Abstract*— We present a differential geometric approach towards the synthesis of cooperative controllers for a team of autonomous surface vehicles transporting a buoyant load. We are interested in cooperative transport of large objects by teams of autonomous surface vehicles (ASVs) operating in marine and littoral environments. We consider the cooperative towing problem where individual ASVs connected to a load via cables must coordinate to transport the load along a desired trajectory. We present a differential geometric approach towards the synthesis of open and closed loop strategies for the team. The main advantage of the proposed strategy is the ability to synthesize agent-level controllers that can simultaneously satisfy all the holonomic and non-holonomic constraints within the system. We validate the approach in both simulations and experiments.**

I. INTRODUCTION

We are interested in cooperative manipulation strategies that enable a team of robots to transport large objects that cannot be transported by single robots alone. Specifically, we are interested in cooperative manipulation and transport of objects in marine and littoral environments where the object is attached to the robots via a cable or a set of cables. Example applications include cooperative towing of large ships, garbage scows, and offshore structures such as oil platforms or wind turbines by a team of autonomous surface vehicles (ASVs). In these applications, a significant challenge lies in the synthesis of control strategies that can simultaneous satisfy all the inherent non-holonomic, *e.g.* vehicles kinematics/dynamics, and holonomic, *e.g.* length of the cable connecting the load to an ASV, constraints within the system while successfully transporting the object along the desired trajectory. Fortunately these problems can be simplified by mapping the cooperative transport problem into an equivalent formation control one.

In general, the addition of a considerable load close to the vehicle's frame often results in increasing the system's inertia and makes the vehicle's response sluggish [1]. One solution for this is to suspend the load from a cable [2], [3]. In [2] the initial lift maneuver is approached as a hybrid system to minimize the swing. Sreenath *et al.* showed how a quad-copter with a cable-suspended load is a differentially flat hybrid system and employs a non-linear geometric controller to control the position and attitude of the quad-copter and the load [3]. The problem of coordinating a large group of small and agile aerial vehicles to haul a larger load is investigated in [4]–[6]. In [4] a kinematic approach is taken. In [5] the equation of motion for the agents is calculated using the Lie group of the transportation matrices, and a geometric feedback controller is designed for each aerial vehicle to follow the generated trajectories. In [6], the control actions are decomposed into parallel and normal components which makes it easier to account for the interactions between the agents. Zhou *et al.* modeled the system as a parallel robot and extended the modeling framework to include vehicles with cable-suspended loads [7]. In all these works, the approach is to first devise a feasible trajectory for the system followed by the design of a controller that enables the system to follow the precomputed trajectory [8].

Different from existing approaches, we formulate the cooperative towing problem as a formation control problem and leverage tools from differential geometry to solve the *control abstraction problem*. Given the kinematics of the robots and a set of robot–load constraints, our objective is to synthesize a control strategy to enable the team to maintain the desired formation while transporting the load along some desired trajectory. We refer to this problem as control abstraction problem. In general, holonomic constraints are imposed on the states of individual robots and formation control objectives give rise to such constraints. On the other hand, non-holonomic constraints are the result of individual vehicle kinematics/dynamics. In our work, we present a framework where these constraints can be combined through the concept of a co-distribution in geometric control [9]–[11]. The result is an approach that enables the design of suitable open and closed-loop control strategies for each member in the team where the resulting trajectories satisfy all the holonomic and nonholomic constraints imposed on the system.

While the proposed theory is general and can be applied to aerial, ground, and marine robots, we focus on the cooperative towing of a floating load by two ASVs. The transport of a floating cargo using a team of ASVs poses unique challenges not encountered with aerial or ground vehicles [12], [13]. For one, the system is not fully controllable since the ASVs are limited in the types of maneuver they can perform. Second, inertia of the vehicles and the load become more significant factors making the system harder to control. In this work, we experimentally validated our proposed strategy to show the robustness of the strategy in the presence of disturbances and model uncertainties [14]–[16].

The rest of this paper is organized as follows: Section II presents some mathematical preliminaries and the formulation of our problem. Section III presents our approach

[1]H. Hajieghrary and D. Kularatne are with the Scalable Autonomous Systems Lab at Drexel University, Philadelphia, PA 19104 USA. {hh449, dnk32}@drexel.edu. [2]M. A. Hsieh is with the GRASP Laboratory at the University of Pennsylvania, Philadelphia, PA 19104 USA. m.hsieh@seas.upenn.edu. We gratefully acknowledge the support of National Science Foundation (NSF) grant CMMI-1462825.

towards the design of feasible solutions to the control abstraction problem. Simulation and experimental results are presented in section IV. We conclude with a discussion of future work in Section IV-B.

II. PROBLEM FORMULATION

A. Preliminaries

We begin with a brief summary of relevant mathematical concepts and model our notation after [9], [17]. Consider a drift-less control-affine system $\mathscr{A}$ of the form

$$\dot{\mathbf{X}} = \sum_{j=1}^{m} \mathbf{F}_j(\mathbf{X})\mathbf{U}_j, \tag{1}$$

where $\mathbf{X} \in \mathbb{R}^n$ denotes the state vector, $\mathbf{F}_j$ are smooth vector fields in $\mathbb{R}^n$, and $\mathbf{U}_j$ are the control inputs. We are interested in synthesizing controllers for systems whose dynamics can be expressed as (1) subject to holonomic and non-holonomic constraints. As such, we use the map $C : \mathcal{U} \in Q \rightarrow \mathbf{0} \in \mathbb{R}^m$ to express the holonomic constraints where Q denotes the configuration manifold of $\mathscr{A}$. From the submersion theorem, if $\mathbf{0}$ is a regular value of C, then $M = C^{-1}(\mathbf{0})$ defines an embedded sub-manifold of co-dimension m [17]. As such, the actual state space of the constrained system is given by the embedded sub-manifold M.

The gradients of the holonomic constraints constitute a set of co-vectors, $dC = \{dc_1, dc_2, ..., dc_m\}$, which annihilates the entire tangent space of M. In other words, if $v_{\gamma(t),p}$ is a tangent vector belonging to T_pM, then $dc_i(v_{\gamma(t),p}) \equiv 0$. For the inverse of this statement to be true the annihilated distribution must be geodesically invariant.

On the other hand, the non-holonomic constraints of the system are represented as a distribution, Δ, on the tangent space of the configuration manifold, TQ. Let $\{e_1, e_2, ..., e_{rank(\Delta)} \mid e_i \in T_pQ\}$ be the local generators of this distribution. If the distribution is regular, *i.e.*, the rank of the local generators of the distribution is constant, then there is a unique annihilating co-distribution on Q associated with the non-holonomic constraints of the system given by $\Lambda = \{\alpha \in T^*Q \mid \alpha(v_{\gamma(t)}) = 0; \forall v_{\gamma(t)} \in \Delta\}$ [18].

Representing the holonomic and non-holonomic constraints of a general dynamical system of the form (1) with the unifying notion of co-distribution gives us the necessary tools to design trajectories for the system that satisfy both sets of the constrains. To do this, we define the intersection of these constraints as a new annihilating co-distribution given by $\Sigma = \{\alpha \in T^*Q \mid \alpha \in dC \bigwedge \alpha \in \Lambda\}$. If this co-distribution is regular, we can find a distribution, $\mathscr{D}$, which will be annihilated by this co-distribution. The existence of such a distribution can be verified in terms of the algebraic rank condition of the matrix representation of the co-distribution. If this distribution is also geodesically invariant, the generators of the distribution will span the tangent space of a trajectory that satisfies the system's dynamics and the holonomic constraints.

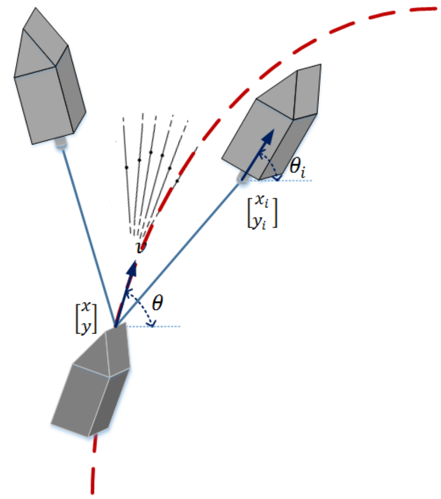


Fig. 1: Schematic of cooperative transport of a buoyant load by two autonomous vehicles. The vehicles tow the load which is connected via cable.

B. Problem Statement

In this work, we are interested in cooperate manipulation for applications such as towing of larger vessels or rafts by teams of autonomous surface vehicles (ASVs) in littoral environments. Fig. 1 shows a team of N ASVs and a load that must be transported along some desired reference trajectory. Let $\mathbf{q_i} = [x_i,\, y_i,\, \theta_i]^T$ denote the pose of the i^{th} robot or ASV and $\mathbf{q}_L = [x,\, y,\, \theta]^T$ be the pose of the load. The vehicle and load kinematics are modeled as

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases}, \qquad \begin{cases} \dot{x}_i = v_i\cos\theta_i \\ \dot{y}_i = v_i\sin\theta_i \\ \dot{\theta}_i = \omega_i \end{cases}. \tag{2}$$

This kinematics correspond to the standard unicycle model and constitutes the set of non-holonomic constraints. The holonomic constraint between an ASV and the load is

$$C_i : \; \left(x - x_i\right)^2 + \left(y - y_i\right)^2 = l_i^2, \tag{3}$$

where l_i denotes the length of the cable connecting the i^{th} vehicle to the load. In this work, we assume the cables do not cross or self-intersect.

From (2), the non-holonomic constraints of the system constitute a distribution on the tangent space of the state manifold is given by

$$\Delta = span\Big\{ \quad \cos\theta\frac{\partial}{\partial x} + \sin\theta\frac{\partial}{\partial y},\; \frac{\partial}{\partial\theta}, \tag{4}$$
$$\cos\theta_i\frac{\partial}{\partial x^i} + \sin\theta_i\frac{\partial}{\partial y^i},\; \frac{\partial}{\partial\theta^i}\Big\}.$$

The co-distribution which annihilates this distribution is of rank two, and can be represented by the following generators

$$\Lambda = \begin{cases} \sin\theta_i dx_i - \cos\theta_i dy_i \\ \sin\theta dx - \cos\theta dy \end{cases}. \tag{5}$$

The differential of the map which constrains the length of the connecting cable is a co-distribution annihilating all vectors on the tangent space of the embedded sub-manifold of the constrained system given by

$$dC_i = \Big\{ \big(x - x_i\big)(dx - dx_i) + \big(y - y_i\big)(dy - dy_i) \Big\}. \quad (6)$$

The two annihilating co-distributions defined by (5) and (6) impose non-holonomic and holonomic constraints on the trajectory of the system. These constraints can be unified into a single co-distribution that annihilates the tangent vector field along any system trajectory such that both sets of constraints are simultaneously satisfied. The components of this co-distribution expressed using the prescribed local chart which is spanned by the basis of $\{dx,\, dy,\, d\theta,\, dx_i,\, dy_i,\, d\theta_i\}$ is given in the following matrix form

$$\Sigma : \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\theta_i & -\cos\theta_i & 0 \\ (x - x_i) & (y - y_i) & 0 & -(x - x_i) & -(y - y_i) & 0 \end{bmatrix}. \quad (7)$$

The matrix representation of the distribution Σ has constant algebraic rank of three which guarantees the existence of a distribution that can be annihilated by the Σ [9]. With some lengthy but straight forward algebraic manipulations, we can compute this distribution which is given by

$$\begin{aligned} \mathscr{D} \;=\; & span\Big\{ \frac{\partial}{\partial\theta^i}, \frac{\partial}{\partial\theta}, h_i \cos\theta \frac{\partial}{\partial x^i} + h_i \sin\theta \frac{\partial}{\partial y^i} + \\ & h \cos\theta_i \frac{\partial}{\partial x} + h \sin\theta_i \frac{\partial}{\partial y} \Big\}, \end{aligned} \quad (8)$$

where $h \triangleq \big(x - x_i\big)\cos\theta + \big(y - y_i\big)\sin\theta$ and $h_i \triangleq \big(x - x_i\big)\cos\theta_i + \big(y - y_i\big)\sin\theta_i$. Any element in the tangent space of the embedded sub manifold of M could be written as linear combination of these vectors. Then, we can write the following equations for the tangent components to any trajectory of the system by linearly combining the basis of the associated space with arbitrary coefficients, ρ, η, and η_i:

$$\begin{cases} \dot{x} = \rho h_i \cos\theta \\ \dot{y} = \rho h_i \sin\theta \\ \dot{\theta} = \eta \end{cases}, \quad \begin{cases} \dot{x_i} = \rho h \cos\theta_i \\ \dot{y_i} = \rho h \sin\theta_i \\ \dot{\theta_i} = \eta_i \end{cases}. \quad (9)$$

By appropriately selecting the values of these coefficients and an admissible set of initial conditions, it is now possible to generate a trajectory that satisfies all the holonomic and non-holonomic constraints of the original system.

We note that a significant advantage of the proposed modeling framework is the ability re-express the constrained system into an equivalent unconstrained one given by (9) where trajectory generation simplifies to suitable values for ρ, η, and η_i. However, since the cables are only modeled by the constraints, the mass,inertia, and drag of the cables are ignored within this framework. As such, explicitly modeling of the cable dynamics is a topic for future work.

III. CONTROLLER SYNTHESIS

In this section we consider both open-loop and closed-loop control for cooperative transport. Since each vehicle is connected to the same load via individual cables, we can first synthesize a virtual controller for the load to specify its desired reference trajectory and then calculate the corresponding control inputs for the ASVs. The open-loop strategy will be executed without realtime feedback of the load's pose [16]. Once the load's trajectory is determined, the ASV control inputs would steer them to satisfy the formation constraints.[1] For the closed-loop strategy, the pose of the load will be used by each ASV to keep the load on an asymptotic approach towards the desired trajectory regardless of the system's initial condition [5], [19].

In this work, the reference trajectory is chosen to be a portion of the curve given by $y_r(t) = a.\tan(\frac{2b}{\pi}x_r(t))$ where a, b are constants. This trajectory is smooth and its curvature can be adjusted to be dynamically feasible for the ASVs and the load. For simplicity, the initial position of the load is chosen to be the origin. The linear speed of the load is assumed to be constant and the orientation and angular velocity of the load is calculated as follows

$$\begin{cases} v_r = v_0 \\ \theta_r = \text{atan2}\left(\frac{dy_r}{dx_r} = \frac{2ab}{\pi}\Big(1 + \tan^2(\frac{2b}{\pi}x_r(t))\Big), 1 \right) \\ \omega_r = \frac{8ab^2 \tan(\frac{2bx_r(t)}{\pi})(1+\tan^2(\frac{2bx_r(t)}{\pi})}{\left((4a^2b^2(tan(\frac{2bx_r(t)}{\pi})^2+1)^2+\pi^2)\right)^{3/2}} v_0 \end{cases} \quad (10)$$

which steer the load along the trajectory given by $y_r(t)$.

A. Open-Loop Control

Given the load trajectory, the ASV trajectories can be obtained using (9). Assuming the cables are taut at all times, one can expect the load to follow its predefined trajectory. The linear speeds for the ASVs are obtained based on the desired speed of the load, *i.e.*, $v_i = \frac{h}{h_i}v_r$, and the angular velocity of each ASV can be chosen arbitrarily.

In this formulation, the ASVs assume the load is always properly constrained, *i.e.*, the two cable are always taut, and thus following the prescribed trajectory. However, in situations where the load or an ASV takes a sharp turn, the cable may become slack due to inertia, causing the load to drift as shown in Fig. 2. To address this, one can further constrain load by adding more robots. However, increasing the team size effectively increases the number of constraints within the system and may significantly reduce the number of feasible solutions. As such, in the absence of feedback, ASVs cannot make the necessary corrective maneuvers to ensure the load stays on the desired trajectory.

B. Closed-Loop Control

To compensate for the inertial effect of the load during transport, we propose a closed-loop control strategy based on the measured pose of the load. This can be achieved by designing an inner control loop to regulate the movement of the load and keep it asymptotically approaching the reference trajectory. We employ extended backstepping control design to achieve this feedback structure, and use the resulting control variables for the load to extract the corresponding ASV trajectories. Consider the extended kinematic model of

[1] The holonomic constraints effectively impose a formation for the fleet of ASVs and the load.

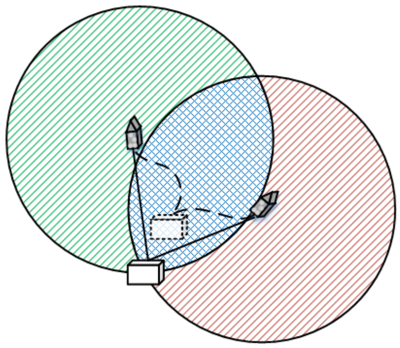


Fig. 2: A buoyant load may drift forward as the result of the inertia or any sudden change in the velocities of the boats resulting in slack cables.

an agent, i.e. the load, where the rates of change of the linear speed and the angular velocity given by:

$$\dot{v} = \frac{1}{M} f, \quad and \quad \dot{\omega} = \frac{1}{J}\tau \tag{11}$$

serve as the control inputs. Here, τ denotes the torque and f represents the force exerted to the system, in this example with the boat. We define the tracking errors as $e_x(t) \triangleq x(t) - x_r(t)$ and $e_y(t) \triangleq y(t) - y_r(t)$. The following steps give the backstepping procedure to compute a controller that can account for the dynamics of the vehicles [20]:

$$\begin{cases} z_{11} = e_x(t) \\ z_{21} = e_x(t) \end{cases} \Rightarrow \begin{cases} z_{12} = \dot{z}_{11} + k_{11} z_{11} \\ z_{22} = \dot{z}_{21} + k_{21} z_{21} \end{cases} \tag{12}$$

$$\Rightarrow \begin{cases} z_{13} = \dot{z}_{12} + z_{11} + k_{12} z_{12} \\ z_{23} = \dot{z}_{22} + z_{21} + k_{22} z_{22} \end{cases} \Rightarrow \begin{cases} \dot{z}_{13} = -k_{13} z_{13} - z_{12} \\ \dot{z}_{23} = -k_{23} z_{23} - z_{22} \end{cases}$$

Following this backstepping chain leads to a system of equations that can stabilize the Lyapunov candidate function given by $V = \frac{1}{2}\sum_{i=1}^{3}\left(z_{1i}^2 + z_{2i}^2\right)$ which guarantees the stability of the tracking controller. Taking the third derivative of the error parameters in the last pair of equations in (12) results in the following system of equation:

$$\begin{aligned} \frac{1}{J} v \sin(\theta)\tau - \frac{1}{m}\cos(\theta)\dot{f} &= x_r^{(3)} + \frac{2f}{m}\omega\sin(\theta) \\ &- \big(k_{11} + k_{13} + k_{11}k_{12}k_{13}\big)(x - x_r) \\ &+ \big(k_{11}k_{12} + k_{11}k_{13} + k_{12}k_{13} + 2\big)\dot{x}_r \\ &+ \big(k_{11} + k_{12} + k_{13}\big)\ddot{x}_r - \big(k_{12}k_{13} + k_{13}k_{11} + 1\big)v\cos(\theta) \\ &+ \big(k_{11} + k_{12} + k_{13}\big)(\omega\frac{f}{m}\cos(\theta) + \omega v\sin(\theta)) \\ &+ \big(k_{11}k_{12} + 1\big)v\sin(\theta) + \omega^2 v\cos(\theta) \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{1}{J} v \cos(\theta)\tau + \frac{1}{m}\sin(\theta)\dot{f} &= y_r^{(3)} - \frac{2f}{m}\omega\cos(\theta) \\ &- \big(k_{21} + k_{23} + k_{21}k_{22}k_{23}\big)(y - y_r) \\ &+ \big(k_{21}k_{22} + k_{23}k_{22} + k_{23}k_{21} + 2\big)\dot{y}_r \\ &+ \big(k_{21} + k_{22} + k_{23}\big)\ddot{y}_r - \big(k_{23}k_{22} + k_{23}k_{21} + 1\big)v\sin(\theta) \\ &- \big(k_{21} + k_{22} + k_{23}\big)(\omega\frac{f}{m}\sin(\theta) + v\omega\cos(\theta)) \\ &- \big(k_{21}k_{22} + 1\big)v\cos(\theta) + \omega^2 v\sin(\theta) \end{aligned}$$

Solving these equations yields the control inputs f and τ and the corresponding v and ω for the load. These parameters are then used to calculate the velocity commands for the ASVs as described in Section III-A. Since the control inputs for the ASVs are defined for the current position of the load, the ASVs will make the necessary corrective maneuvers to return the load to the predefined trajectory in the presence of drift or disturbances.

IV. RESULTS

A. Simulation Results

We present simulation results for the open-loop case. Rather than arbitrarily select the angular velocities for the ASVs, we consider the speeds of the vehicles. Geometrically, if the linear speeds of the load and a boat are equal they have to move parallel to each other to satisfy the formation constraints [21], [22]. The following equation shows the relationship between the load speed and the ASVs speeds

$$\begin{aligned} \dot{v}_i &= \frac{\dot{h}}{h_i} v_r - \frac{h}{h_i^2}\dot{h}_i v_r \\ &= \frac{1}{h_i}\big(v_r - v_i\cos(\theta - \theta_i) - h_\theta\omega\big)v_r \\ &- \frac{h}{h_i^2}\big(v_r\cos(\theta - \theta_i) - v_i - h_{\theta_i}\omega_i\big)v_r, \end{aligned} \tag{14}$$

where $h_{\theta_i} = \frac{\partial h_i}{\partial \theta_i}$, $h_\theta = \frac{\partial h}{\partial \theta}$, and the linear speed of the ASV is assumed constant. In (14) the angular velocity of the ASV, ω_i, can be chosen such that v_i achieves its target value of v. Thus,the angular velocity of each ASV can determined as follows:

$$\begin{aligned} \omega_i = &-\frac{1}{h_i h_{\theta_i}}\big(v - v_i\cos(\theta - \theta_i) - h_\theta\omega\big) \\ &- \frac{h}{h_i^2 h_{\theta_i}}\big(v\cos(\theta - \theta_i) - v_i\big) + \frac{1}{h_{\theta_i}\omega_i}\big(\dot{v} - K(v_i - v)\big) \end{aligned} \tag{15}$$

and results in a simple first order linear dynamics with a time constant of K on the ASV velocity of the form

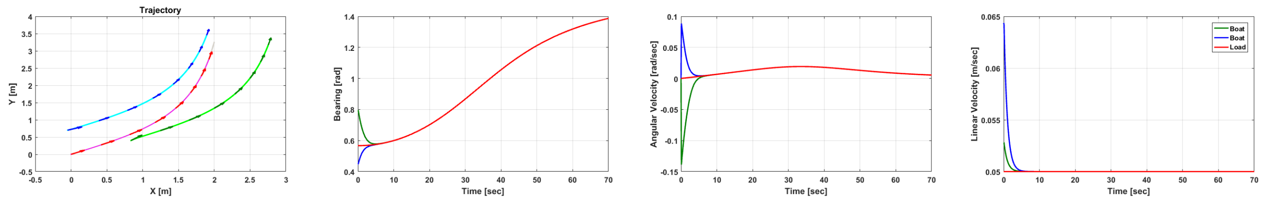
$$(\dot{v}_i - \dot{v}) - K(v_i - v) = 0. \tag{16}$$

This strategy simultaneously regulates the speed and orientation of each ASV.

Fig. 3 shows the simulation result of this strategy for two boats towing a load. At the beginning since the directions of the boats are not aligned with the initial direction of the trajectory of the load, the boats moved at high speeds to ensure the constraints are always satisfied along their trajectories. Over time, ASV speeds converge asymptotically to the load's intended speed.

B. Experimental Results

We validate the proposed control strategy using the Multi-Robot Coherent Structure Testbed (mCoSTe). The mCoSTe is an indoor laboratory testbed that consists of a $3m \times 3m \times 1m$ water tank (Fig. 4) and a fleet of micro Autonomous Surface Vehicles (mASVs). The mASVs are differential vehicles equipped with a micro-controller board, XBee radio module, and an inertial measurement unit (IMU). Each



(a) Trajectory (b) Bearings (attitude) (c) Angular Velocities (d) Linear Velocities

Fig. 3: Simulation results for the proposed kinematic closed-loop control of two boats hauling a floated load. After a transient state both boats turn to move parallel to the load. the objective of the designed control algorithm for the boats is to regularize their linear velocity to track the designed velocity of the load.

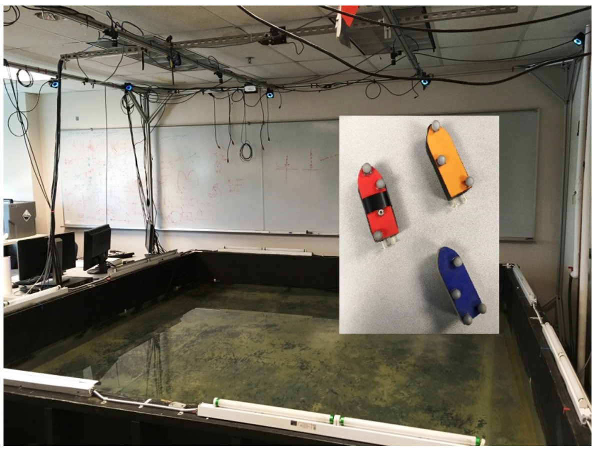


Fig. 4: Multi-Robot Tank covered with motion capture localization cameras and the micro Autonomous Surface Vehicles (mASVs). These differential drive model vessels are equipped with a micro-controller board, XBee radio module, and an inertial measurement unit (IMU). The boats tow the floating load connected via massless cables.

mASV is approximately $12cm$ long with a mass of about $45g$. Localization for the mASVs and the load is provided by an external motion capture system.

Fig. 5 shows the experimental results for the proposed open-loop strategy. The trajectories of the boats are designed with respect to the desired trajectory of the load without feedback on the load's position and orientation. The strategy assumes the load is secured, the tow cables are taut strapped, and the load does in fact follow the presumed trajectory.

Fig. 6 shows the experimental results for the closed-loop strategy based on the position and orientation of the load. While initial pose of the load deviates from reference trajectory, the closed-loop strategy ensures the ASVs steer the load such that it settles along its reference trajectory.

In both the open-loop and closed-loop experiments, the linear speeds and the orientation of each ASV converge to linear speeds and the orientation of the load, respectively. The oscillations in the vehicles' and load's bearings and speeds are the result of the low-level controllers running onboard the ASVs. Using larger vehicles would most likely dampen these unnecessary oscillations and result in much smoother control commands. A video of the experiments is available at https://youtu.be/Okw6cGmsw3A [23].

CONCLUSION AND FUTURE WORK

In this paper we addressed the cooperative transport problem for a team of ASVs towing a buoyant load. The problem is first mapped to an equivalent formation control problem. We presented an approach towards the synthesis of open and close loop strategies for the team such that the resulting trajectories for each robot that satisfy all holonomic and non-holonomic constraints while transporting the load along a desired path. We validated the strategy on a team of micro ASVs and show the applicability of the proposed control approach in a real world application. In this work, the holonomic constraints are only imposed by the length of the cables attaching the load to the individual ASVs. For future, we would like to include robot-robot constraints as well as more complex load trajectories. Specifically, we are interested in further validating the proposed strategy in situations where there is significant slack in the cable connecting the load and the robot and in the presence of an external flow field. Lastly, we are interested in extending the proposed strategy to three dimensions and validating it on aerial and ground vehicles.

REFERENCES

[1] I. Palunko, P. Cruz, and R. Fierro, "Agile load transportation : Safe and efficient load manipulation with aerial robots," *Robotics Automation Magazine, IEEE*, vol. 19, no. 3, pp. 69–79, Sept 2012.

[2] P. Cruz, M. Oishi, and R. Fierro, "Lift of a cable-suspended load by a quadrotor: A hybrid system approach," in *American Control Conference (ACC), 2015*, July 2015, pp. 1887–1892.

[3] K. Sreenath, T. Lee, and V. Kumar, "Geometric control and differential flatness of a quadrotor uav with a cable-suspended load," in *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, Dec 2013, pp. 2269–2274.

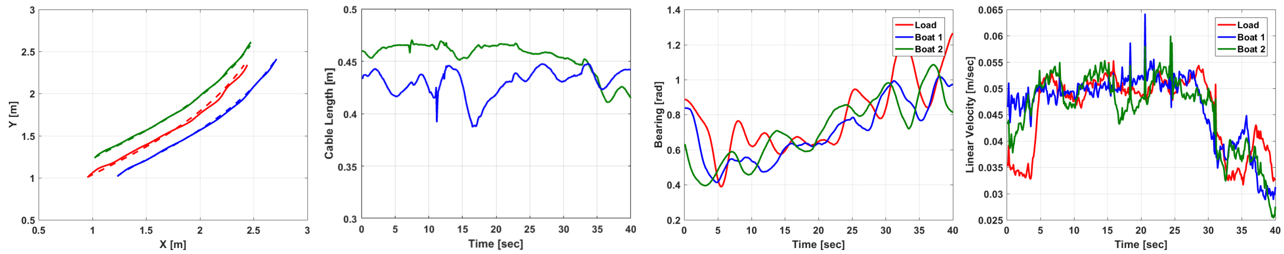
[4] N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Autonomous Robots*, vol. 30, no. 1, pp. 73–86, 2011. [Online]. Available: http://dx.doi.org/10.1007/s10514-010-9205-0

[5] G. Wu and K. Sreenath, "Geometric control of multiple quadrotors transporting a rigid-body load," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, Dec 2014, pp. 6141–6148.

[6] T. Lee, "Geometric control of multiple quadrotor uavs transporting a cable-suspended rigid body," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, Dec 2014, pp. 6155–6160.

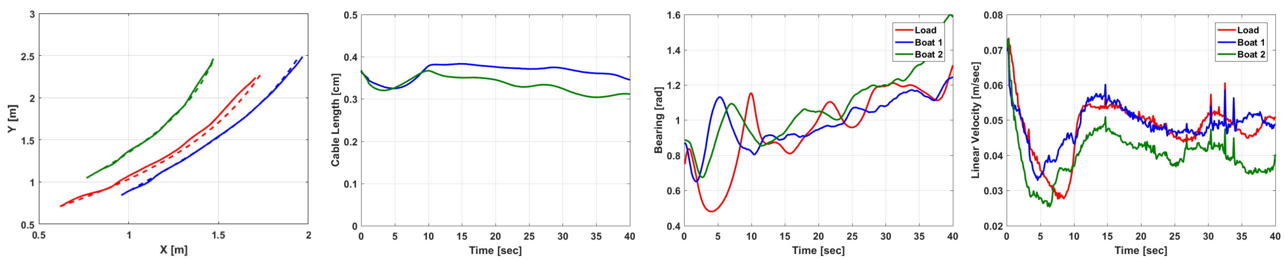
[7] X. Zhou, C. P. Tang, and V. Krovi, "Analysis framework for cooperating mobile cable robots," in *Robotics and Automation (ICRA), 2012 IEEE International Conference on*, May 2012, pp. 3128–3133.

[8] K. Klausen, T. I. Fossen, T. A. Johansen, and A. P. Aguiar, "Cooperative path-following for multirotor uavs with a suspended payload," in *Multi-Conference on Systems and Control, 2015 IEEE*, Sep. 2015.



(a) Trajectory (b) Bearing (attitude) (c) Angular Velocities (d) Linear Velocities

Fig. 5: The Experimental results for the proposed *Open-Loop* kinematic control. Although the boats are following their predefined trajectory, the load is struggling to overcome its inertia.The dotted line indicates the desired position of the load and the boats, while the solid line is the position of them read by the motion capture system.



(a) Trajectory (b) Bearing (attitude) (c) Angular Velocities (d) Linear Velocities

Fig. 6: The Experimental results for the proposed kinematic *Closed-Loop* control algorithm. The trajectory of the load is designed through a feedback algorithm. The inner loop gets feedback from the position of the load to correct its course. The outer loop is designed similar to previous case to regulate the direction of the boats. The load starts the tracking with an orientation other than the initial angle of tangent to the desired trajectory. Then it initially exhibits a maneuver to finally settles down on the reference course. The dotted line indicates the desired position of the load and the boats, while the solid line is the position of them read by the motion capture system.

[9] F. Bullo and A. Lewis, *Geometric Control of Mechanical Systems: Modeling, Analysis, and Design for Simple Mechanical Control Systems*, ser. Texts in Applied Mathematics. Springer New York, 2004. [Online]. Available: https://books.google.com/books?id=jg7VumxOoe4C

[10] P. Tabuada, G. Pappas, and P. Lima, "Motion feasibility of multi-agent formations," *Robotics, IEEE Transactions on*, vol. 21, no. 3, pp. 387–392, June 2005.

[11] A. Ailon and I. Zohar, "Control strategies for driving a group of nonholonomic kinematic mobile robots in formation along a time-parameterized path," *IEEE/ASME Transactions on Mechatronics*, vol. 17, no. 2, pp. 326–336, April 2012.

[12] J. M. Esposito, "Decentralized cooperative manipulation with a swarm of mobile robots," in *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Oct 2009, pp. 5333–5338.

[13] J. Esposito, M. Feemster, and E. Smith, "Cooperative manipulation on the water using a swarm of autonomous tugboats," in *2008 IEEE International Conference on Robotics and Automation*, May 2008, pp. 1501–1506.

[14] C. F. L. Thorvaldsen and R. Skjetne, "Formation control of fully-actuated marine vessels using group agreement protocols," in *2011 50th IEEE Conference on Decision and Control and European Control Conference*, Dec 2011, pp. 4132–4139.

[15] C. Tam and R. Bucknall, "Cooperative path planning algorithm for marine surface vessels," *Ocean Engineering*, vol. 57, pp. 25 – 33, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0029801812003472

[16] Y. Liu and R. Bucknall, "Path planning algorithm for unmanned surface vehicle formations in a practical maritime environment," *Ocean Engineering*, vol. 97, pp. 126 – 144, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0029801815000165

[17] J. Lee, *Introduction to Topological Manifolds*, ser. Graduate texts in mathematics. Springer, 2000. [Online]. Available: https://books.google.com/books?id=5LqQgkS3--MC

[18] A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and R. M. Murray, "Nonholonomic mechanical systems with symmetry," *Archive for Rational Mechanics and Analysis*, vol. 136, no. 1, pp. 21–99, 1996. [Online]. Available: http://dx.doi.org/10.1007/BF02199365

[19] S. Park, J. Deyst, and J. P. How, "Performance and lyapunov stability of a nonlinear path following guidance method," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 6, pp. 1718–1728, 2007.

[20] H. Hajieghrary and M. Ani Hsieh, "Dynamic adaptive robust backstepping control design for an uncertain linear system," *Journal of Dynamic Systems, Measurement, and Control*, vol. 138, no. 7, pp. 071004–1–071004–8, 2016. [Online]. Available: http://dx.doi.org/10.1115/1.4033019

[21] D. Maithripala and S. Jayasuriya, "Feasibility considerations in formation control: Phantom track generation through multi-uav collaboration," in *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*, Dec 2008, pp. 3959–3964.

[22] H. Hajieghrary and S. Jayasuriya, "Guaranteed consensus in radar deception with a phantom track," in *Dynamic Systems and Control Conference*, Dec 2013, p. V002T20A005.

[23] H. Hajieghrary, D. Kularatnel, and M. A. Hsieh. IROS 2017: Trajectory planning for transportation of a buoyant load. Youtube. [Online]. Available: https://youtu.be/Okw6cGmsw3A